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CARDINAL POINTS OF THICK LENS & THICK LENS FORMULA

Cardinal points of a thick lens:- In Gaussian optics, there are three pairs of points for characterizing an optical system. These six points are known as cardinal points. These all cardinal points lie on the optical (principal) axis of the optical system.

There are six cardinal points namely two principal points, two focal points and two nodal points for a thick lens for characterizing the thick lens. Planes passing through these six points and perpendicular to the optical (principal) axis of the lens, are known as cardinal planes.

The cardinal points and the cardinal planes are intrinsic properties of a particular optical system (Here thick lens) and they are used to determine the image forming properties of the system. If these are known, then we can find the image of any object without making a detailed study of the passage of the rays through the system.

These all six cardinal points lie on the principal axis of a thick lens which may be located as follows.

* principal points of a thick lens:-

In shown figure, CD is a thick lens bounded by two spherical surfaces separated by a distance t.

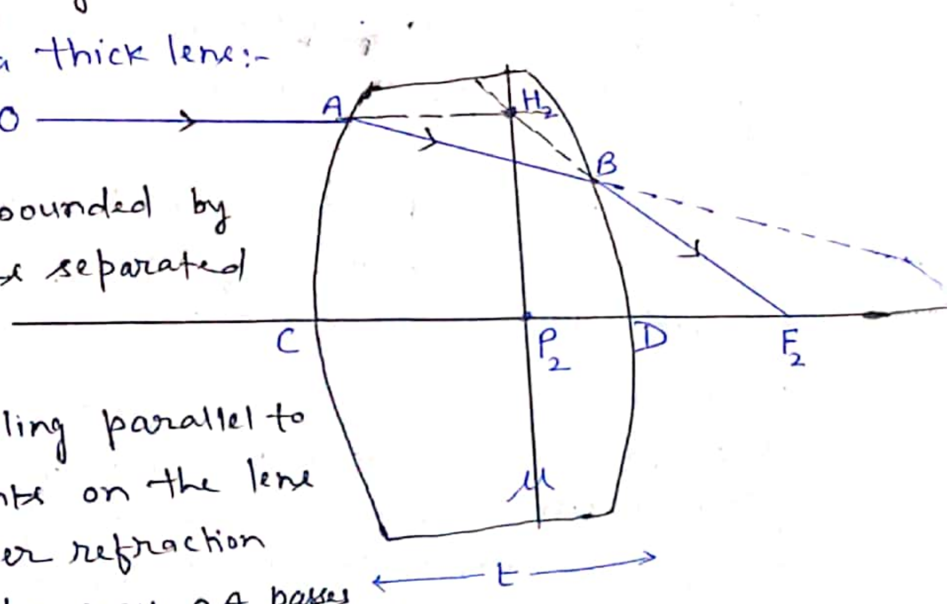


Fig- 1

Let a ray OA travelling parallel to principal axis, incidents on the lens at point A and after refraction through the lens, the ray OA passes through the focal point F_2 along BF_2 in

The image space of the lens as shown in fig-1. From fig-1, it is clear that the incident ray OA and emergent ray BF_2 appear to intersect each other at point H_2. Now we can explain the refraction of the incident ray OA through the thick lens in terms of single refraction at a plane H_2P_2 passing through the point of intersection H_2 and perpendicular to the principal axis. Such a plane H_2P_2 is known as second principal plane and the point of intersection P_2 of second principal plane H_2P_2 with the principal axis, is known as second principal point in the image space.

Similarly, we can locate first principal H_1P_1 and first principal point P_1 in the object space as shown in fig-2.

Let a ray TQ travelling parallel to principal axis, incidents on the lens at point Q and after refraction through the lens, the ray TQ passes through the focal point F_1 along SF_1. H_1 is the point of intersection of incident ray TQ and emergent ray SF_1. A plane H_1P_1 passing through H_1 and perpendicular to principal axis of the lens is known as first principal plane and the point of intersection P_1 of first principal plane H_1P_1 with the principal axis, is known as first principal point in the object space.

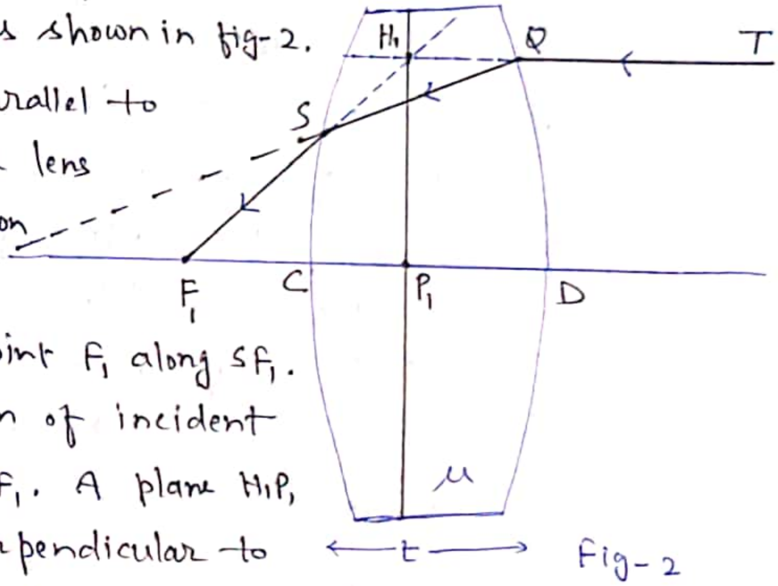


Fig-2

Thus P_1 and P_2 are first principal point (in object space) and second principal point (in image space) respectively & H_1P_1 and H_2P_2 are first principal plane and second principal plane respectively.

* Focal points of thick lens:- The first focal point F_1 of a thick lens is a point on the principal axis of the lens such as a beam of light passing through F_1 becomes parallel after refraction through the lens as shown in fig-2. The second focal point F_2 of the thick lens is a point on the principal axis of the lens such as a beam of light travelling parallel to the principal

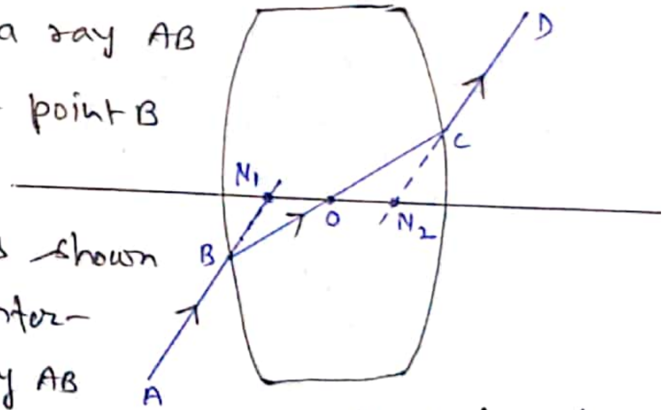
axis of the lens after refraction through the lens, passes through F_2. The planes passing through focal points F_1 and F_2 and perpendicular to principal axis are known as first focal plane and second focal plane respectively.

The main features (properties) of focal planes is that the rays starting from a point in focal plane in object space correspond to a set of conjugate parallel rays in image space. Similarly a set of parallel rays in object space correspond to a set of rays intersecting at a point in focal plane in image space.

The distance F_1P_1 of the first focal point F_1 from the first principal point P_1, is known as first focal length f_1 and distance F_2P_2 of the second focal point F_2 from the second principal point P_2 is known as second focal length f_2.

f_1 and f_2 are also known as focal lengths of the lens in object space and image space respectively. If refractive index of the medium on two sides of the lens be same then $f_1 = f_2$ numerically.

* Nodal points of thick lens :- If a ray AB is incident on a thick lens at point B such that it passes through the centre O of the lens as shown in figure, then the point of intersection N_1 of the incident ray AB with principal axis, is known as first nodal point in object space and the point of intersection N_2 of the emergent ray CD with principal axis, is known as second nodal point in image space. The planes passing through nodal points N_1 and N_2 and perpendicular to principal axis, are known as first nodal plane and second nodal plane respectively.



Note :- If refractive index of medium on two sides of the lens be same then nodal points and principal points are coincident i.e, $N_1 = P_1$ & $N_2 = P_2$

The principal planes are planes where all refractions are assumed to occur whereas nodal planes are planes where refraction does not take place.

USE OF CARDINAL POINTS IN STUDY OF IMAGE FORMATION BY A THICK LENS:

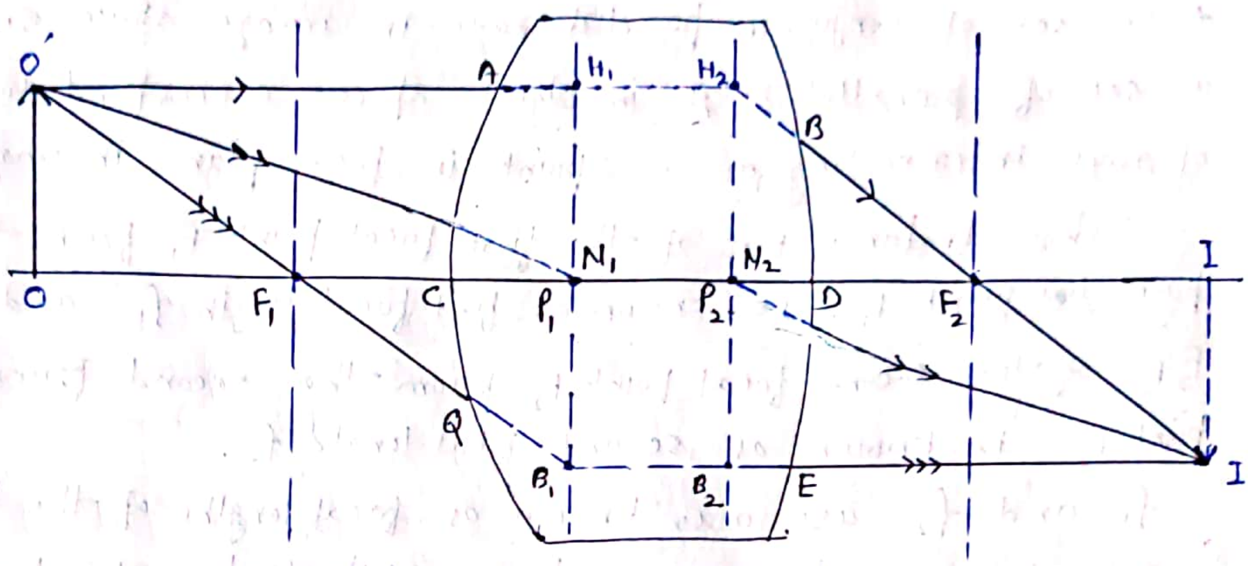


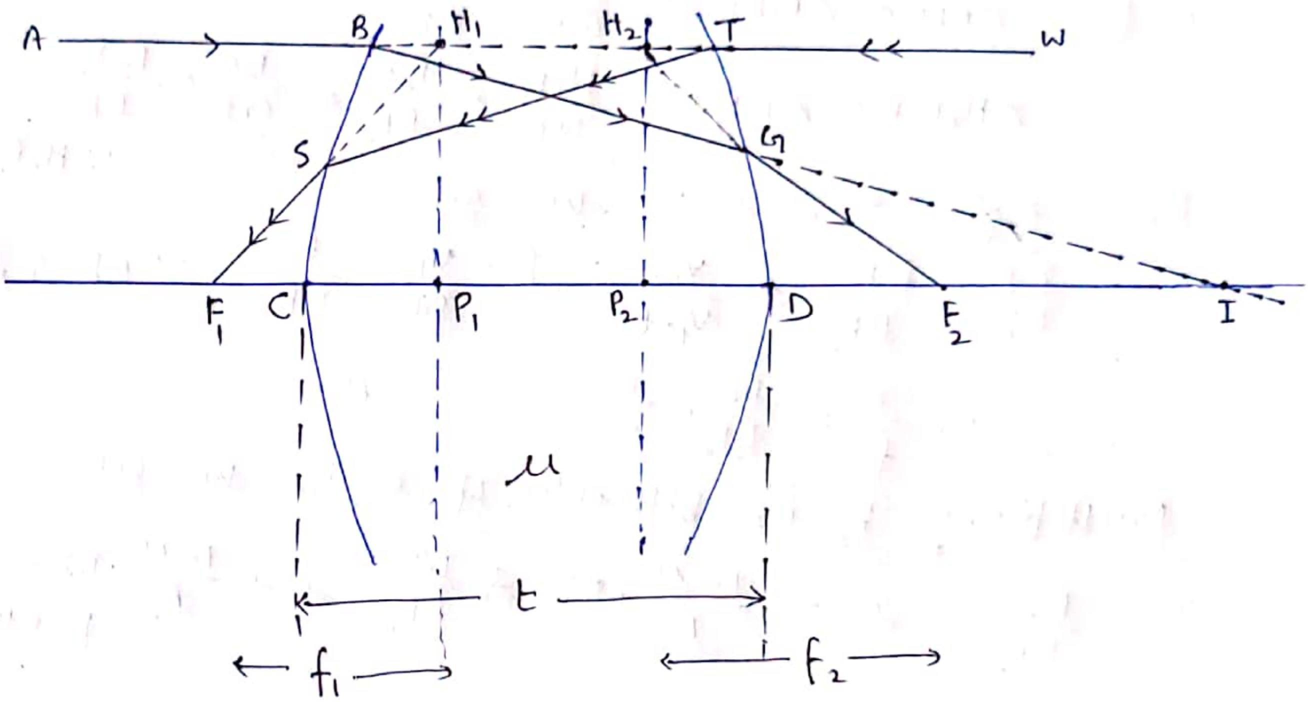
Fig-3:

The cardinal points and cardinal planes of a thick lens have been shown in fig-3.

The formation of image of an object OO' by a thick lens by using cardinal points have been shown in fig-3. An object OO' is placed on the axis of a thick lens. The paraxial ray O'A travelling parallel to axis, incident on the lens at A and after refraction through the lens, it passes through second focal point F_2. Second paraxial ray $O'N_1$ starting from O' passes through the first nodal point N_1 and after refraction through the lens, it emerges at along N_2I'. Third paraxial ray $O'F_1Q$ starting from O' passes through first focal point F_1 and after refraction through the lens, it emerges at E along EI' parallel to the principal axis. The three emergent rays meet at I' which is image of O'. Now draw a line I'I perpendicular to the principal axis. Thus II' will represent image of the object OO'.

THICK LENS FORMULA OR EQUATION

Let CD is a thick lens of refractive index μ and thickness t. It consists of two spherical surfaces BC and TD of radii of curvatures R_1 and R_2 respectively. Medium on either side of the lens is air. Let H_1P_1 and H_2P_2 be first and second principal planes. $F_1P_1 = f_1$ and $F_2P_2 = f_2$ are first and second focal lengths of the lens.



Here $f_1 = f_2 = f$ (numerically) because medium on either side of the lens is air.

Now a ray AB parallel to the principal axis coming from infinity, incidents on the lens at B on its first surface and after refraction through first surface BC, it meets the principal axis at I along BGI in the absence of the second surface but in the presence of the second surface TD, it is further refracted and it meets at F_2 (second focal point) on the axis.

For the first surface BC, $u=\infty$, $v = CI = v_1$ (let), $R = R_1$, $\mu_1 = 1$ for air, $\mu_2 = \mu$ for lens

using refraction formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{\mu}{v_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \Rightarrow \frac{\mu}{v_1} - 0 = \frac{\mu-1}{R_1} \Rightarrow \frac{\mu}{v_1} = \frac{\mu-1}{R_1}$$

$$\Rightarrow v_1 = \frac{\mu R_1}{\mu - 1} \quad \text{——— (1)}$$

Now I behaves as an object and F_2 as image for the second surface TD.

For second surface TD, $u = DI$, $v = DF_2$, $R = R_2$, $\mu_1 = \mu$ for lens, $\mu_2 = 1$ for air

Using refraction formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1}{DF_2} - \frac{\mu}{DI} = \frac{1-\mu}{R_2} \quad \text{——— (2)}$$

From fig, $\Delta BIC \sim \Delta GID \Rightarrow \frac{BC}{GD} = \frac{CI}{DI}$ ——— (3)

$\Delta H_2F_2P_2 \sim \Delta GF_2D \Rightarrow \frac{H_2P_2}{GD} = \frac{P_2F_2}{DF_2} \Rightarrow \frac{BC}{GD} = \frac{P_2F_2}{DF_2}$ ——— (4) $\because H_2P_2 = BC$

From eqn (3) and (4), we get

$$\frac{CI}{DI} = \frac{P_2F_2}{DF_2} \Longrightarrow \frac{v_1}{DI} = \frac{f}{DF_2} \qquad \because CI = v_1,\ P_2F_2 = f$$

$$\Rightarrow \frac{1}{f} = \frac{DI}{DF_2} \cdot \frac{1}{v_1} \quad \text{——— (5)}$$

Multiplying by DI bothsides of eqn (2), we get

$$\frac{DI}{DF_2} - \mu = \frac{1-\mu}{R_2} \cdot DI \Rightarrow \frac{DI}{DF_2} = \mu + \frac{1-\mu}{R_2} \cdot DI \quad \text{——— (6)}$$

put in eqn (5)

We get

$$\frac{1}{f} = \left[\mu + \frac{1-\mu}{R_2} \cdot DI\right] \cdot \frac{1}{v_1}$$

$$\Rightarrow \frac{1}{f} = \left[\mu + \frac{1-\mu}{R_2}(v_1 - t)\right] \cdot \frac{1}{v_1} \quad \text{——— (7)} \quad \because DI = CI - CD = v_1 - t$$

Using eqn (1) in eqn (7), we get

$$\frac{1}{f} = \left[\mu + \frac{1-\mu}{R_2}\left\{\frac{\mu R_1}{\mu-1} - t\right\}\right] \frac{\mu-1}{\mu R_1}$$

$$= \left[\mu - \frac{\mu-1}{R_2} \cdot \left\{\frac{\mu R_1 - (\mu-1)t}{\mu-1}\right\}\right] \cdot \frac{\mu-1}{\mu R_1}$$

$$= \frac{\mu R_2 - \mu R_1 + (\mu-1)t}{R_2} \cdot \frac{\mu-1}{\mu R_1}$$

$$\boxed{\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1 R_2}\right]}$$

It is thick lens formula.

Ques:- Explain cardinal points of thick lens. Deduce thick lens formula.

Ques:- Explain cardinal points of thick lens. Discuss their use in the study of image formation by such a lens.